The Importance of Math to Quantum Theory

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"Some physicists would prefer to come back to the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist independently of whether we observe them. This however is impossible" (Herbert 31).

This "objective real world," or the classical theory of physics, describes only two things--matter and fields; it is the study of "matter and energy and their interactions in the fields of mechanics, acoustics, optics, heat, electricity, magnetism, [and] radiation" (Webster-Merriam, 866). Ordinary particles studied within classical theory follow observable patterns or laws that can be measured; however, there are some invisible particles that violate the laws of classical theory. Here, classical theory fails to explain certain phenomena such as those of light and electrons; it has been replaced by the newer quantum theory.

Quantum theory, however, has one drawback. Unlike the classical theory of physics, in which knowledge is gained by directly observing the motion of objects, quantum theory cannot be observed. Because of the invisibility of quantum theory, some other tool must be utilized to study elementary particles; to do this, the only practical tool is mathematics.

Quantum theory, the study of elementary particles, begins with the simple idea of an electron residing on the perimeter of an atom. From experiments with gold foil, in which the negatively charged electrons bounced off positively charged nuclei in the center of gold atoms, scientists as Ernest Rutherford and Niels Bohr had concluded that electrons must occupy certain levels of energy around the nuclei. Therefore, "[t]o move from one energy level to another, an electron must gain or lose the right amount of energy. . . A *quantum* of energy is the amount of energy required to move an electron from its present energy level to the next higher one" (Wilbraham 325).

How did scientists determine the existence of electron energy levels if they were invisible? According to the Heisenberg uncertainty principle, named after its discoverer Werner Heisenberg in 1927, "if you measure position [of the electron] accurately, you must sacrifice an accurate knowledge of [its] momentum. . . [The principle] prevents anyone from resolving the quantum reality question via the clear light of experiment" (Herbert 68-69). The Heisenberg uncertainty principle states that no one can define the exact structure or position of an electron; if physical tools or forces are used, they will fail. However, although direct measurements cannot be made, mathematics can be used to stitch together available data into usable, quantitative patterns.

Using math, ideas too complex to be described in words may be represented by symbols. In 1926, the Austrian physicist Erwin Schrodinger used the "new quantum theory to write and solve a mathematical equation describing the location and energy of an electron in a hydrogen atom" (Wilbraham 326). The modern description of the electrons in atoms, known as the quantum mechanical model, is used widely in chemistry--it comes from the solution to the Schrodinger equation and is primarily mathematical, having few analogies in the visible world. From the Schrodinger equation, cloud-shaped regions where electrons reside have been described, energy levels of chemical elements have been determined, and all chemical phenomena of the periodic table have been explained. None of these advances could have been accomplished without the Schrodinger equation.

"Quantum mechanics does not predict a single definite result for an observation. Instead, it predicts a number of different possible outcomes and tells us how likely each of these is" (Hawking 55). To predict the outcomes of any physical event, probability must be used. This form of mathematics "introduces an unavoidable element of unpredictability or randomness into science," but it also allows scientists to infer judgment on an otherwise invisible, unreachable subject" (Herbert 56). Probability lists all the possible outcomes for an event. Although scientists may have never seen the event taking place, probability offers scientists various explanations for the occurrence.

The polarization, or "bending" of light, employs probability. Light, a quantum of energy packaged in photons, can be polarized either diagonally or horizontally. Either polarization has a fifty percent probability of occurring. If a material such as metal-coated glass was placed between both light beams, four possible polarizations may be obtained: diagonal-left and diagonal-right, horizontal and vertical. Each original polarization had a fifty percent chance of occurring; because each beam was divided in half again, the probability of either of the latter four polarizations occurring is twenty-five percent. Because of this effect, simple math can explain how metal-tinted sunglasses deflect bright light and ultraviolet radiation from the eyes.

Probability is also a rule used to describe elementary particles themselves. While classical theory deals with visible, macroscopic objects and forces, quantum physics focuses on ". . . the elemental scale, at the level which elementary particles interact to form atoms and produce forces" (Wilson). Every electron possesses an intrinsic property called spin, or its angular rotation, as it travels the atom. According to the Pauli exclusion principle, a rule of probability, ". . . two electrons must have opposite spins. . . spin is a quantum property of electrons and may be *clockwise* or *counterclockwise*" (Wilbraham 331). Therefore, for two electrons to pair together, one must have positive spin and the other a negative spin. From the concept of spin, electrons configure about the atom and determine the atom's chemistry; spin is the most basic property of elementary particles.

Besides probability, equations that relate different ideas together are always used. In 1924, Louis de Broglie, a French graduate student, predicted that all matter exhibits wavelike motions. It is almost impossible to imagine matter moving as a wave. A still rock, for example, produces no visible motion, but quantum theory states that it does. De Broglie proved the wavelike motion of matter by combining two equations representative of either matter or waves into one equation stating that the matter equation was equal to the wave equation.

The first equation explains that energy (e) is equal to Planck's constant (h), named after Max Planck's unifying value of joules (units of energy) per Hertz (unit of frequency), multiplied by the frequency (f) of the wave.

The second equation states that wavelength (λ) is equal to the speed of light, meters/second, divided by the frequency (f) of the wave.

In both equations, the value for frequency is shared. Therefore, the speed of light divided by wavelength is equal to the energy divided by Planck's constant.

Written another way, the above equation shows that wavelength, a property "unique unto waves," is equal to Planck's constant divided by momentum (mass times volume), a property "unique unto matter" (Wilson).

The mathematics necessary to understand quantum theory is not derived from probability or equations, but from the need for a measuring tool. Through probability and equations, however, scientists can deduce measurements, data, and create conclusions based on their observations. In the visible world, quantum theory has no representations. Elementary particles are so small, they must be subjected to high levels of energy for them to separate from each other; even then, they fly free only for a short time. The most powerful microscope today, the scanning-tunnelling microscope, can "see" the surface of atoms--the electron cloud--but no further. No one has ever seen the nucleus within the atom, or the twenty or so major different particles created by the atom. The only observations that can be made are on the effects of these particles, such as a massless neutrino's tracks through contained water more than a thousand feet underground, or the patterns of light resulting from collisions within a cyclotron.

However, math can be used to predict and justify quantum theory easier, with undisputed results. Without having to build a cyclotron, also called a linear particle accelerator, mathematics allows easy explanation of quantum theory to students. Expensive tools such as cyclotrons and scanning-tunnelling microscopes are impractical tools for the general study of quantum theory. Also, mathematical equations represent real ideas in a form that is easier to manipulate. When describing quantum theory, many ideas must be discussed at once; equations and formulas may be solved two or three at a time, handling as many numbers as needed to represent the data. Because math is an orderly, unambiguous process, correct conclusions may be raised each time.

Finally, as with the discovery of all new territory, there is often debate over how newly discovered sciences work. Unlike people, math is unbiased. Simple mathematics is enough to establish a claim for a theory; if another mathematical operation refutes it, then the theory is proven wrong. The Heisenberg uncertainty principle was originally a proposed atomic model, but was changed because its principles apply to two-dimensional instead of three-dimensional objects--because all matter and waves are three-dimensional, math proved Heisenberg wrong.

Since the beginning of quantum theory in the early twenties, mathematics has been the greatest tool to physicists in this field. It has become the primary means of expressing and acquiring information about the theory. Beginning with invisible electrons residing on the outer limits of atoms, mathematics, especially probability and quantum equations, has advanced the theory to its present state--quantum theory can explain any physical occurrence with a simple manipulation of math. "No development of modern science has had a more profound impact on human thinking than the advent of quantum theory," and the advent of the mathematics to support it (Herbert 16).

Outline

I. Introduction

II. What is Quantum Theory?

- A. Beginning of quantum theory, atomic energy levels
- B. The Heisenberg Uncertainty Principle

III. The Importance of Math

- A. The Schrodinger equation
- B. Probability
  1. Light polarization as a measure of probability
  2. Use of probability in describing elemental particles
- C. Importance of Quantum Equations
- D. Lack of a means of measurement
- E. Advantages of using math as a tool
- F. Math used in the justification of quantum theory

IV. Conclusion

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